Approximate Quadrature Measures on Data–Defined Spaces

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An important question in the theory of approximate integration is to study the conditions on the nodes $x_{k,n}$ and weights $w_{k,n}$ that allow an estimate of the form

$$\sup_{f\in\mathcal{B}_\gamma}\left|\sum_k w_{k,n}f(x_{k,n})-\int_{\mathbb{X}} f d\mu^*\right|\le cn^{-\gamma}, \qquad n=1,2,\cdots,$$

where $\mathbb{X}$ is often a manifold with its volume measure μ^*, and $\mathcal{B}_\gamma$ is the unit ball of a suitably defined smoothness class, parametrized by γ. In this talk, we present some of our recent results in this direction in the context of a quasi-metric, locally compact, measure space $\mathbb{X}$ with a probability measure μ^*. We show that quadrature formulas exact for integrating the so called diffusion polynomials of degree $< n$ satisfy such estimates. Without requiring exactness, such formulas can be obtained as a solutions of some kernel-based optimization problem. We discuss the connection with the question of optimal covering radius. Our results generalize in some sense many recent results in this direction.