Orthogonal polynomials, zeros and electrostatics

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In this talk, we will analyze the electrostatic interpretation of the zeros of two sequences of polynomials related to orthogonal polynomials on the real line and orthogonal polynomials on the unit circle, respectively. The first one appears in the framework of the spectral theory of second order linear differential equations with rational functions as coefficients. Their polynomial eigenfunctions are said to be exceptional orthogonal polynomials and they have known an increasing interest in the recent time. I will focus the attention on the so called exceptional Laguerre and Jacobi polynomials (see [1]). The second one deals with para-orthogonal polynomials associated with differentiable weights supported on the unit circle (see [3]). A survey of these contributions will be given.

The basic ingredient is the fact that their zeros are simple and from the second order differential equation that those polynomials satisfy you get a direct interpretation in terms of a normal electrostatic equilibrium (see [2]). Some open problems will be formulated for other examples of non standard orthogonal polynomials as those related to Sobolev inner products.

References

[1] A. P. Horvath, *The electrostatic properties of zeros of exceptional Laguerre and Jacobi polynomials and stable interpolation*. J. Approx. Theory *194* (2015), 87-107.

[2] F. Marcellán, A. Martínez-Finkelshtein, P. Martínez-González, *Electrostatic models for zeros of polynomials: old, new, and some open problems*. J. Comput. Appl. Math. 207 (2007), no. 2, 258-272.

[3] B. Simanek, *An electrostatic interpretation of the zeros of paraorthogonal polynomials on the unit circle*. SIAM J. Math. Anal. **48** (2016), no. 3, 2250-2268.