Hyperuniformity on the sphere

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The concept of hyperuniformity had been introduced by S. Torquato and F. Stillinger to measure regularity of distributions of infinite particle systems in $\mathbb{R}^d$. An infinite particle system $X \subset \mathbb{R}^d$ is called hyperuniform, if the variance (with respect to the thermodynamic limit) of the number of points in a **large** ball is smaller than "usual":

$$\mathbb{V}\#(X \cap B(\mathbf{x}, R)) = \mathcal{O}(R^{d-1}) \text{ for } R \to \infty.$$

Notice that this variance is of order R^d for Poisson point processes.

We generalise this concept to the sphere and the torus by considering sequences of **finite** point sets $(X_N)_N$ (with $\#X_N = N$). The phenomenon of a "smaller than usual" variance of the point counting function is then be observed for geodesic balls with $N\mathrm{vol}(B_R) \to \infty$ but $\mathrm{vol}(B_R) \to 0$. We will discuss several examples of hyperuniform sequences of point sets.

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