An optimal bound for designs on real algebraic varieties

Ujué Etayo

Universidad de Cantabria

Spherical $t$-designs defined in the $d$-dimensional sphere were initially defined by P. Del-sarte, J. M. Goethals and J.J. Seidel. Since then, spherical designs have gained attraction in different areas of mathematics, see [1] for a review. In [2] Bondarenko, Radchenko and Viazovska proved the conjecture of Korevaar and Meyers, stating that for all $N \geq C t^d$ there exists a spherical $t$-design in $S^d$ consisting of $N$ points.

There have been made some generalizations of designs in compact symmetric spaces of rank 1 and in different kinds of schemes (Johnson, Hamming,...). In this talk we present a generalization of the result of Bondarenko, Radchenko and Viazovska into a more general setting.

Let $M \in \mathbb{R}^n$ be a smooth compact algebraic variety of dimension $d$, then we define a $t$-design in $M$ as a set of points $\{x_1, ..., x_N\} \in M$ such that

$$\int_M P(x) d\mu_M(x) = \frac{1}{N} \sum_{i=1}^N P(x_i)$$

for all algebraic polynomials in $n$ variables, of total degree at most $t$, where $\mu_M$ is the normalized Lebesgue measure.

Then, we state that for each $N \geq C_M t^d$ there exists a $t$-design in $M$ consisting of $N$ points, where $C_M$ is a constant depending on the manifold.

Joint work with J. Marzo and J. Ortega-Cerdà.

References


