An optimal bound for designs on real algebraic varieties

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Spherical t-designs defined in the d-dimensional sphere were initially defined by P. Delsarte, J. M. Goethals and J.J. Seidel. Since then, spherical designs have gained attraction in different areas of mathematics, see [1] for a review. In [2] Bondarenko, Radchenko and Viazovska proved the conjecture of Korevaar and Meyers, stating that for all $N \geq Ct^d$ there exists a spherical t-design in $\mathbb{S}^d$ consisting of N points.

There have been made some generalizations of designs in compact symmetric spaces of rank 1 and in different kinds of schemes (Johnson, Hamming,...). In this talk we present a generalization of the result of Bondarenko, Radchenko and Viazovska into a more general setting.

Let $\mathcal{M} \in \mathbb{R}^n$ be a smooth compact algebraic variety of dimension d, then we define a t-design in $\mathcal{M}$ as a set of points $\{x_1, ..., x_N\} \in \mathcal{M}$ such that

$$\int_{\mathcal{M}} P(x) d\mu_{\mathcal{M}}(x) = \frac{1}{N} \sum_{i=1}^{N} P(x_i) \tag{1}$$

for all algebraic polynomials in n variables, of total degree at most t, where $\mu_{\mathcal{M}}$ is the normalized Lebesgue measure.

Then, we state that for each $N \geq C_{\mathcal{M}} t^d$ there exists a t-design in $\mathcal{M}$ consisting of N points, where $C_{\mathcal{M}}$ is a constant depending on the manifold.

Joint work with J. Marzo and J. Ortega-Cerdà.

References

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