

Strongly regular graphs and Borsuk's conjecture

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In 1933 Karol Borsuk conjectured that every closed set in \mathbb{R}^n of diameter 1 could be partitioned into $n + 1$ parts of smaller diameters. The conjecture was disproved in 1993 by Kahn and Kalai. In particular, their construction gives counterexamples for Borsuk's conjecture for $n = 1325$ and for all $n > 2014$. Until recently the best known result was that Borsuk's conjecture is false for all $n \geq 298$.

We will show how to use the Euclidean representation of $G_2(4)$ strongly regular graph to construct a two-distance set consisting of 416 points on the unit sphere in the dimension 65 which cannot be partitioned into 83 parts of smaller diameter.

References:

1. J. Kahn, G. Kalai, *A counterexample to Borsuk's conjecture*, Bull. Amer. Math. Soc., 29 (1993), 60-62.
2. A. Bondarenko, *On Borsuk's conjecture for two-distance sets*, Discrete and Computational Geometry, 51 (2014) 509-515.